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Control of solidification boundary in continuous casting by asymmetric cooling and mold offset

ROBERT SIEGEL

National Aeronautics and Space Administration, Lewis Research Center, Cleveland, Ohio 44135, U.S.A.

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NOMENCLATURE

- A — dimensionless parameter and dimensionless length, $a\bar{u}\rho\lambda/k(t_f-t_c) = a/\gamma$
- a — half-width of slab ingot
- b — offset of mold; $B = b/\gamma$
- $\Delta h, \Delta k$ — grid sizes in potential plane
- I, J — grid indexes in potential plane
- k — thermal conductivity of solidified material
- n — normal to interface; $N = n/\gamma$
- S — dimensionless coordinate along solidification interface
- t — temperature
- $\bar{u}$ — casting velocity of ingot
- x, y — Cartesian coordinates; $X = x/\gamma$, $Y = y/\gamma$.

Greek symbols

- γ — length scale parameter, $k(t_f-t_c)/\bar{u}\rho\lambda$
- θ — angle between interface normal and y-axis
- λ — latent heat of fusion per unit mass of solid
- ρ — density of solid
- Φ — potential function, $(t_f-t)/(t_f-t_c)$; $\Phi_i = (t_f-t_i)/(t_f-t_c)$
- Ψ — heat flow function orthogonal to Φ
- ω — over-relaxation factor
- $\tilde{\nabla}^2$ — $\partial^2/\partial X^2+\partial^2/\partial Y^2$.

Subscripts

- c — at boundary cooled to t_c
- f — at solidification temperature
- i — at boundary cooled to t_i
- s — at solidification interface.

INTRODUCTION

THIS NOTE is a further development of the analyses in references [1] and [2]. During solidification, the shape of the solid–liquid interface is important as it influences the resulting crystal structure. In continuous casting, where an ingot is being withdrawn from a mold, the solidification interface (which is a 'free' boundary) is regulated by the cooling conditions and mold shape. In [1] and [2], two analytical methods were given that yielded exact solutions for the free-boundary shapes. It was shown that it is much more convenient to obtain results by an inverse-type of solution wherein the physical coordinates are dependent variables of orthogonal temperature and heat-flow functions. This type of solution will be further developed here to obtain solidification-interface shapes for more complex situations wherein both the ingot cooling and mold geometry are asymmetric.

ANALYSIS

This analysis starts with some of the ideas in the second analytical method of [2] and provides a procedure to deal with further generalized boundary conditions for ingot casting. Figure 1(a) shows the ingot being cast at velocity $\bar{u}$ from an insulated mold where $\partial t/\partial x = 0$ at the mold boundaries. The liquid metal and solidification interface are at t_f. One side of the ingot is cooled to t_i and the other side is cooled to $t_c < t_i$. In the solid at the interface, $k\partial t/\partial n = \rho\bar{u}\lambda \cos\theta$, and within the solid, $\nabla^2 t = 0$. As defined in the Nomenclature, the temperatures are incorporated into a dimensionless potential Φ such that at the solidification interface $\Phi = 0$, at one ingot side $\Phi = \Phi_i$ where $0 < \Phi_i \leqslant 1$, and at the other side $\Phi = 1$ (see Fig. 1(b)). In the solid $\tilde{\nabla}^2\Phi = 0$ and at the unknown interface $-\partial\Phi/\partial N = \cos\theta$.

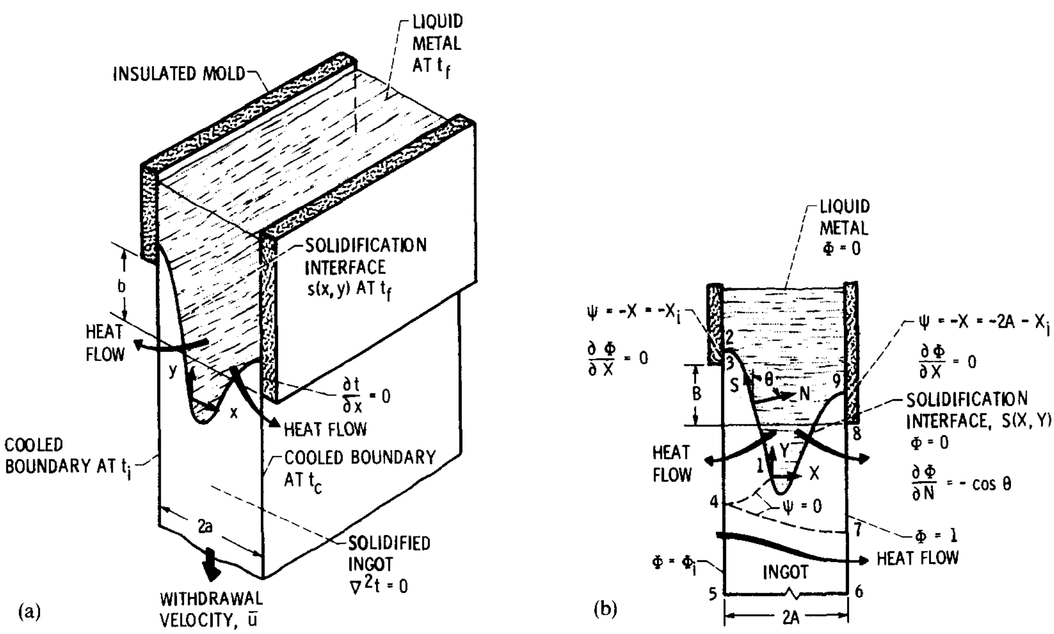

FIG. 1. Slab ingot with unequally cooled sides being withdrawn from offset mold in continuous casting. (a) Physical conditions. (b) Geometry and boundary conditions in dimensionless physical plane.

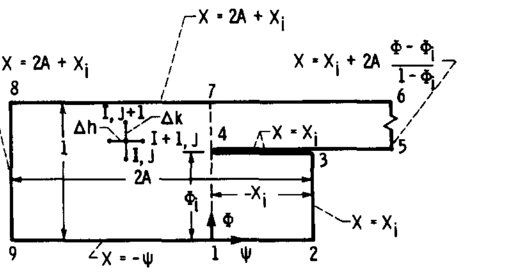

FIG. 2. Ingot mapped into potential plane; within region, $\partial^2X/\partial\Psi^2+\partial^2X/\partial\Phi^2=0$.

To avoid dealing with the unknown boundary in the physical plane, the problem will be inverted as discussed in [1] to solve for $X(\Psi,\Phi)$, where Ψ is a heat-flow function orthogonal to Φ. Then, as discussed in [2], Y can be found as the conjugate harmonic-function of X; the coordinates of the interface are thus obtained. As shown in Fig. 2, in a potential plane the ingot maps into a generalized rectangular region with a cut. The lower boundary $\widehat{912}$ is the solidification interface at $\Phi=0$, and the upper boundary $\widehat{678}$ is the ingot side cooled to $\Phi=1$. The cut along Φ_i is the other cooled side of the ingot. The two sides $\widehat{98}$ and $\widehat{23}$ are the insulated mold boundaries; these are normal to the constant temperature lines and thus are lines of constant Ψ. The dashed lines in Fig. 1(b) show schematically the dividing heat-flow lines. In addition to energy transfer from the interface to the left and right ingot sides, there is heat flow across the lower part of the ingot since $t_i>t_c$.

Within the region of Fig. 2, X is a harmonic function of Ψ and Φ and can be found by solving a Dirichlet problem. The origin in Fig. 1(b) is placed at the dividing streamline along the interface. Since this location is unknown, the X_i (a negative value) is an intermediate quantity in the solution, and will be related to the governing parameters: the dimensionless width $2A$, the temperature ratio Φ_i and the mold offset B. If an X_i value is chosen, then the boundary value is known along $\widehat{2345}$ in Fig. 2. Since A is a specified parameter, then $X=2A+X_i$ is known along $\widehat{6789}$. The strip ending at $\widehat{56}$ is extended far enough beyond $\widehat{23}$ (generally $A/2$) so that the Φ lines in it become parallel and evenly spaced as dictated by physical conditions in the lower part of the ingot; then along $\widehat{56}$, X is a linear function of Φ.

The only remaining boundary condition in Fig. 2 is the solidification interface $\widehat{912}$ along which $-\partial\Phi/\partial N=\cos\theta$. The Ψ and Φ are orthogonal so that the Cauchy–Riemann equations yield at the interface $\partial\Phi/\partial N=-\partial\Psi/\partial S$, and $\partial\Phi/\partial S=\partial\Psi/\partial N=0$ since Φ is constant along S. Then along the interface $\partial\Psi/\partial X=(\partial\Psi/\partial S)(\partial S/\partial X)+(\partial\Psi/\partial N)(\partial N/\partial X)=\cos\theta/(-\cos\theta)=-1$, so that along the lower boundary in Fig. 2, $X=-\Psi$.

The region in Fig. 2 was divided into a rectangular grid and the X values obtained by successive overrelaxation using the relation,

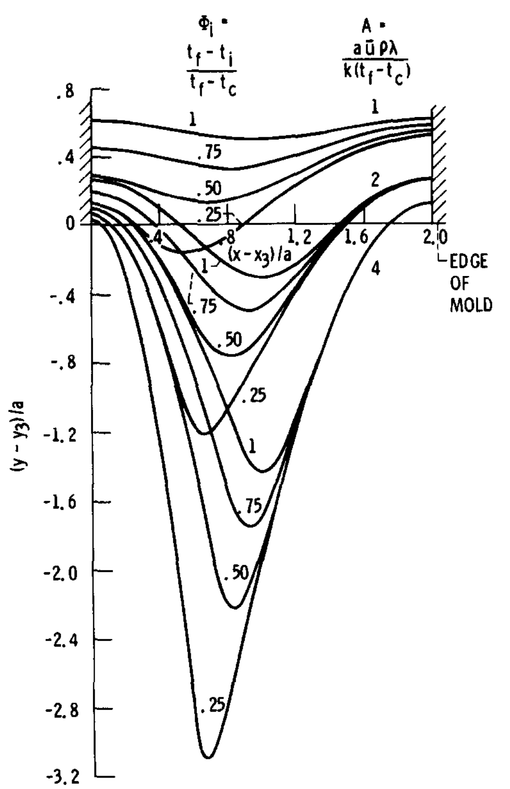

FIG. 3. Effect of unequal ingot side temperatures on solidification interface shapes for three values of A.

using the Cauchy–Riemann relations, $\partial X/\partial\Psi=\partial Y/\partial\Phi$ and $\partial X/\partial\Phi=-\partial Y/\partial\Psi$. Then along the interface

$$Y_s(\Psi)-Y_9=-\int_{\Psi_9}^{\Psi}\left.\frac{\partial X}{\partial\Phi}\right|_{\Phi=0}d\Psi. \tag{2}$$

Three-point one-sided differences were used for the derivative, and integration was carried out with the trapezoidal rule. Accuracy was checked using other differentiation and integration schemes. The interface location within the mold, and the mold offset were found by integrating $\partial X/\partial\Psi$ along $\widehat{98}$ and $\widehat{23}$. Since X along the interface is known as $X_s(\Psi)=-\Psi$, the $Y_s(\Psi)$ from Equation (2) determines the free-boundary shape.

The total range of Ψ on the interface is $2A$. Hence the fraction of the total solidification energy that flows out of the left-hand side of the ingot in Fig. 1(b) is $-X_i/2A$. These values are in Table 1 as a function of the three governing parameters.

$$X(I,J)_{\text{new}}=\left\{\frac{X(I+1,J)+X(I-1,J)+(\Delta h/\Delta k)^2[X(I,J+1)+X(I,J-1)]}{2[1+(\Delta h/\Delta k)^2]}-X(I,J)\right\}\omega+X(I,J). \tag{1}$$

Alternatively, conformal mapping can be used, but the mapping transformations become quite complex. Values were computed for $A=1,2,3,4$ and for $\Phi_i=0.25$, 0.50, and 0.75. The Δh and Δk were usually in the range of 0.025. With an overrelaxation factor $\omega=1.85$, about 350 iterations were generally needed; typically, each case required about 6 s for $A=1$ and 25 sec for $A=4$ on an IBM 370. The mold offset, calculated in the following, is determined by the X_i value so that a few cases for different X_i had to be computed for each A and Φ_i combination.

After the X values are found, the Y values are obtained by

RESULTS AND DISCUSSION

The analysis shows that the ingot can be mapped into a generalized-type of rectangular region with a cut, and that values of $X(\Psi,\Phi)$ are known on all sides of the region. The $X(\Psi,\Phi)$ can then be obtained as a harmonic function within the region (Dirichlet problem), and $Y(\Psi,\Phi)$ values found as the conjugate harmonic function of $X(\Psi,\Phi)$. The resulting coordinates of the solidification interface are given in Figs 3 and 4 for various values of the parameters, A, Φ_i and b/a.

Figure 3 shows results for a symmetric mold (zero-offset). A

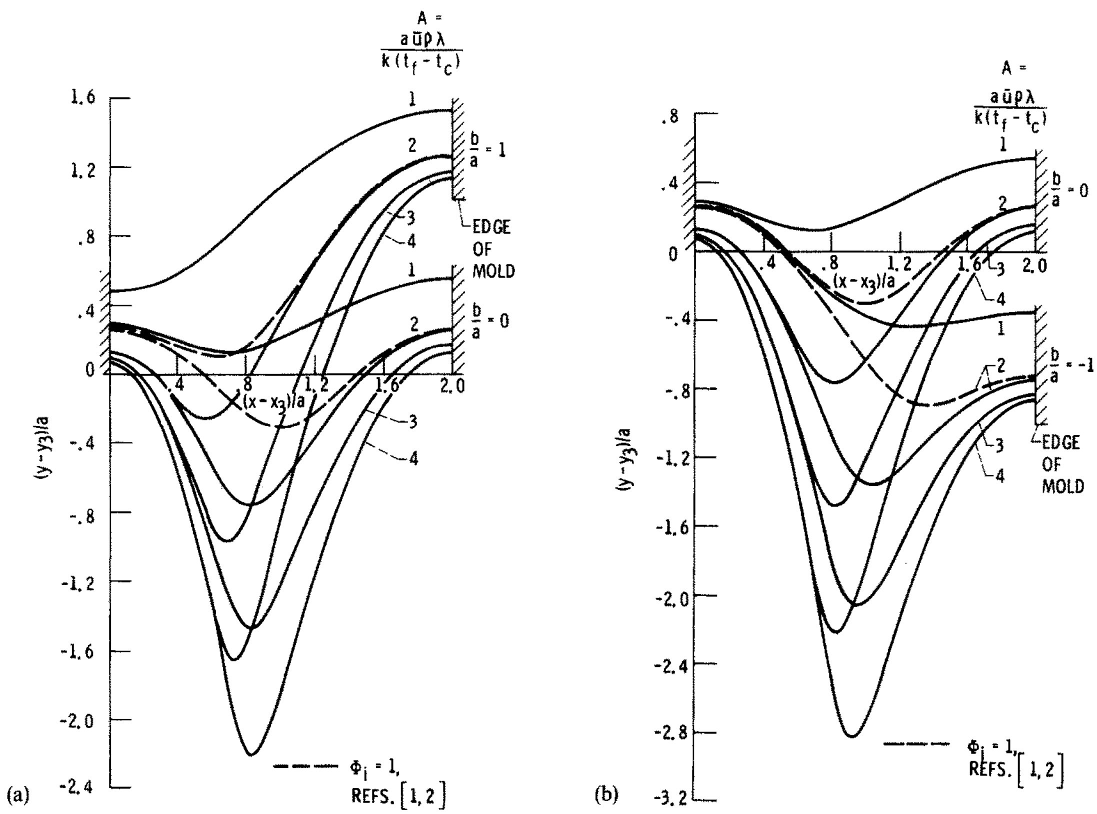

FIG. 4. Effect of mold offset on interface shapes as a function of A for $\Phi_i = 0.5$. (a) Mold offset, $b/a = 0, 1$. (b) Mold offset, $b/a = 0, -1$.

Table 1. Fraction of solidification energy transferred through left side of ingot, $-X_i/2A$

		$b/a = 0$		$b/a = -1$	$b/a = +1$
A	$\Phi_i =$ 0.25	0.50	0.75	0.50	0.50
1	0.209	0.271	0.355	0.434	0.101
2	0.281	0.352	0.412	0.437	0.259
3	0.299	0.373	0.431	0.432	0.314
4	0.309	0.385	0.440	0.429	0.340

group of curves is given for each of three values of the parameters $A = a\bar{u}\rho\lambda/k(t_f - t_c)$. The A contains the ingot casting velocity $\bar{u}$, and as $\bar{u}$ is increased the interface dips further down within the mold. This provides shorter heat-flow paths from the interface to the cooled sides so that the solidification energy can be conducted away. For $\Phi_i = (t_f - t_i)/(t_f - t_c) = 1$ there is equal cooling at both ingot sides and the interfaces are symmetric. When $\Phi_i < 1$ the left side of the ingot is being cooled less than the right side. The interface becomes asymmetric with the thicker region of solid along the right side as the result of the larger cooling. Since the average temperature of the sides is less than when $\Phi_i = 1$, the interface must dip further down into the mold as Φ_i is reduced for a fixed casting rate (fixed A). This provides shorter heat-flow paths required to transfer away the fixed amount of solidification energy with a lower average temperature-difference between the solidification interface and ingot sides.

In Fig. 1(b) the upper branch of the dividing streamline $\Psi = 0$ intersects the interface at $X = 0$, and it divides the total solidification energy into the portions flowing to the left and right ingot sides. Since at the ends of the interface (points 2 and 9) the Ψ values are, respectively, $-X_i$ and $-2A - X_i$, the fraction of solidification energy transferred through the left side is $-X_i/2A$. These values are in Table 1. For a symmetric interface $-X_i/2A = 1/2$, the values in the Table move toward 1/2 as A and Φ_i increase, as this provides the most nearly symmetric curves as shown in Fig. 3.

Figure 4 gives results for two mold offsets, $b/a = \pm 1$, as compared with the symmetric mold. The solid curves are all for $\Phi_i = 0.5$; the dashed curves for $\Phi_i = 1.0$ are included for comparison for $A = 2$ (from [1] and [2]). A positive offset of the right mold side moves the minimum point of the interface toward the left side. For $\Phi_i = 0.5$ there is less cooling on the left side and this also moves the minimum point toward the left. These results, along with those in Fig. 3, show how the solidification interface can be controlled by asymmetries in both mold geometry and cooling of the ingot sides.

REFERENCES

1. R. Siegel, Cauchy method for solidification interface shape during continuous casting, *J. Heat Transfer* **105**, 667–671 (1983).
2. R. Siegel, Solidification interface shape for continuous casting in an offset mold—two analytical methods, *J. Heat Transfer* **106**, 237–240 (1984).